A NEW LOOK AT DECOMPOSITION OF TURBULENCE FORCING FIELD AND THE STRUCTURAL RESPONSE

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Abstract

Measured cross-spectrum of a turbulence field usually shows some decay in the statistical correlation in addition to convection at a characteristic velocity. Under such a random excitation the computation of structural response statistics becomes much more tedious than that which would be the case if the turbulence were convected without decay; i.e. convected as a frozen-pattern. It is shown in this paper that a decaying turbulence can be decomposed into frozen-pattern components thus permitting a simpler way to calculate the structural response. The procedure so devised also provides a relationship whereby the measured input spectra can be incorporated. For illustration the theory is applied to an infinite beam which is backed on one side by a fluid-filled cavity and is exposed on the other side by the turbulence excitation. The effect of the free stream velocity is also taken into consideration.

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Introduction

From the standpoint of structural response calculation the simplest mathematical model for an atmospheric or boundary-layer turbulence is one that is statistically homogeneous in space and is convected in a given direction as a frozen pattern. The second part of the assumption is known as Taylor's hypothesis which often results in tremendous computational savings. In some cases it may be the key assumption making the problems solvable.

However, experimental measurements of real turbulences invariably show that spatial decays do exist in the cross-spectra or cross correlation functions. Such decays are indicative of the change in turbulence patterns as they move down-stream. Therefore, results obtained from a frozen-pattern analysis are just crude estimates which stand to be improved when a better method becomes available.

The primary objective of this paper is to show that a decaying turbulence

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can be constructed by superposing infinitely many frozen-pattern components with random amplitudes and convected at different velocities. Then the structural response can also be similarly superposed with each component corresponding to one frozen-pattern component in the forcing field.

To focus attention on the essentials, the discussion in this paper will be restricted to one-dimensional space coordinate. We shall begin by reviewing some basic relations for the frozen-pattern case as building blocks for the later superposition. Then the proposed superposition will be developed. Finally, as an example, the theory will be applied to an infinite beam under the excitation of a supersonic boundary-layer turbulence.

Frozen-Pattern Turbulence

An example of one-dimensional structure exposed to the excitation of a turbulent pressure field is depicted in Fig. 1. The x coordinate frame is stationary with respect to the undeformed structure, and it will be referred to as the fixed frame in the sequel. If the pressure is truly of a frozen type and is convected at a constant velocity U_c in the positive x-direction, then it is a random function of $x - U_c t$. Such a random function can be expressed as a Fourier-Stieltjes integral as follows:

$$p(x - U_c t) = \int_{-\infty}^{\infty} e^{i(\omega t - kx)} \, dF(k) \qquad (1)$$

where the frequency ω and wave number k are related to the convection speed U_c as $\omega/k = U_c$. It is known from the random process theory that

$$E\,\{dF(k_1)\; dF^*(k_2)\} = S_p(k_1)\; \delta(k_1 - k_2)\; dk_1\; dk_2 \qquad (2)$$

where E{ } represents the ensemble average, an asterisk denotes the complex conjugate, and $S_p(k)$ is the wave-number spectrum in a coordinate frame moving at the velocity U_c (referred to as the moving frame in the sequel).

The cross-correlation function $E\{p(x_1 - U_c t_1)\, p(x_2 - U_c t_2)\}$ of the pressure, referred to the fixed frame, can be calculated simply by use of Eqs. (1) and (2). This function, denoted by R_p, depends only on $\xi - U_c\tau$ where $\xi = x_1 - x_2$ and $\tau = t_1 - t_2$, and it is related to the moving-frame wave-number spectrum $S_p(k)$ as follows:

$$R_p(\xi - U_c\tau) = \int_{-\infty}^{\infty} e^{ik(U_c\tau - \xi)}\, S_p(k)\, dk \tag{3}$$

If a Riemann-Fourier transform is taken of Eq. (3) we obtain the fixed-frame frequency cross-spectrum of p:

$$\begin{aligned} \Phi_p(\xi, \omega) &= \frac{1}{2\pi} \int_{-\infty}^{\infty} R_p(\xi - U_c\tau)\, e^{-i\omega\tau}\, d\tau \\ &= \frac{1}{|U_c|}\, S_p\!\left(\frac{\omega}{U_c}\right) e^{-i(\omega/U_c)\,\xi} \end{aligned} \tag{4}$$

Equation (4) shows that the fixed-frame frequency cross-spectrum of a frozen-pattern turbulence has a special form where ξ appears only in the imaginary exponent. This equation also provides a simple formula to convert from $S_p(k)$ to $\Phi_p(\xi, \omega)$.

Conversely, to convert from $\Phi_p(\xi, \omega)$ to $S_p(k)$:

$$S_p(k) = |U_c|\, \Phi_p(0, kU_c) \tag{5}$$

Evaluated at $\xi = 0$ the cross-spectrum $\Phi_p(\xi, \omega)$ reduces, of course, to the usual spectrum.

We emphasize that Eqs. (4) and (5) are valid only if the turbulence is

strictly of a frozen pattern, and is convected at speed U_c.

Equation (1) suggests that structural response to a frozen-pattern turbulence can be constructed from a fundamental solution where the excitation is just a convected sinusoidial pattern of unit amplitude. Thus, let $H(x, k) \exp(i\omega t)$ be the steady-state solution for

$$\mathcal{L}\{ H(x, k) \exp(i\omega t) \} = \exp [i(\omega t - kx)] \tag{6}$$

where, symbolically, $\mathcal{L}$ represents a linear operator in x and t, pertaining to the dynamic problem at hand. Of course, this solution must satisfy all the necessary boundary conditions. Then the solution to

$$\mathcal{L}\{ w(x, t) \} = p(x - U_c t) \tag{7}$$

after reaching stochastic stationarity, may be expressed as

$$\begin{aligned} w(x, t) &= \int_{-\infty}^{\infty} H(x, k) \exp(i\omega t)\, dF(k) \\ &= \int_{-\infty}^{\infty} H(x, k) \exp(iU_c kt)\, dF(k) \end{aligned} \tag{8}$$

It follows that the cross-correlation function of the structural response is

$$\begin{aligned} &E\{ w(x_1, t_1)\, w(x_2, t_2)\} \\ &= \iint_{-\infty}^{\infty} H(x_1, k_1)\, H^*(x_2, k_2)\, e^{iU_c(k_1 t_1 - k_2 t_2)}\, S_p(k_1)\, \delta(k_1 - k_2)\, dk_1\, dk_2 \\ &= \int_{-\infty}^{\infty} H(x_1, k)\, H^*(x_2, k)\, e^{iU_c k(t_1 - t_2)}\, S_p(k)\, dk \end{aligned} \tag{9}$$

As expected, this correlation function is dependent only on $t_1 - t_2$. If it is desired to calculate this correlation function in the frequency domain, we may substitute Eq. (5) into Eq. (9) and change $U_c k$ to ω:

$$E\{w(x_1, t_1)\, w(x_2, t_2)\}$$

$$= \int_{-\infty}^{\infty} H(x_1, \omega/U_c)\, H^*(x_2, \omega/U_c)\, e^{i\omega(t_1 - t_2)} \Phi_p(0, \omega)\, d\omega \qquad (10)$$

In terms of the input and output spectra the relations are extremely simple and illuminative; they are:

in the wave number domain:

$$S_w(x_1, x_2; k) = H(x_1, k)\, H^*(x_2, k)\, S_p(k) \qquad (11)$$

in the frequency domain:

$$\Phi_w(x_1, x_2; \omega) = H(x_1, \omega/U_c)\, H^*(x_2, \omega/U_c)\, \Phi_p(0, \omega) \qquad (12)$$

When $x_1 = x_2$ these formulas reduce to those for the usual spectra, and they have the same form as the well-known result for a single degree of freedom system in the random vibration theory. The simplicity is a direct consequence of the frozen-pattern assumption.

Decaying Turbulence

Measured frequency cross-spectra for real turbulences with respect to a fixed frame of reference have the general form of

$$\overline{\Phi}_p(\xi, \omega) = \overline{\Phi}_p(0, \omega)\, \psi(\xi)\, \exp(-i\omega\xi/U_c) \qquad (13)$$

where ψ is an even non-negative definite function of ξ, having an absolute maximum equal to one at $\xi = 0$ and approaching to zero at large absolute values of ξ. This form is sometimes attributed to Corcos[1]. A number of researchers have reported curve-fitted results for $\overline{\Phi}_p(0, \omega)$ and $\psi(\xi)$. For representative works we cite the papers by Bull[2], Willmarth and Wooldridge[3], and Maestrello, et.al.[4] Implicit in Eq. (13) is that a real turbulence is not a frozen one.

To obtain a theoretical spectrum consistent with Eq. (13) we propose the following representation for a general turbulence pressure

$$p(x, t) = \int_{-\infty}^{\infty} \hat{p}(x - ut)\, dG(u) \tag{14}$$

Equation (14) implies that $p(x, t)$ is a superposition of infinitely many frozen-pattern components, each having a random amplitude $dG(u)$ and a convection velocity u. Such velocities can assume either positive or negative values. Of course, each frozen-pattern component can, again, be decomposed into sinusoids. Thus,

$$p(x, t) = \iint_{-\infty}^{\infty} e^{i(u\beta t - \beta x)}\, dF(\beta, u)\, dG(u) \tag{15}$$

and its fixed-frame correlation is

$$\begin{aligned} &E\{ p(x_1, t_1)\, p(x_2, t_2)\} \\ &= \iiiint_{-\infty}^{\infty} e^{i(u_1\beta_1 t_1 - u_2\beta_2 t_2) - i(\beta_1 x_1 - \beta_2 x_2)} \\ &\quad E\{ dF(\beta_1, u_1)\, dF^*(\beta_2, u_2)\, dG(u_1)\, dG^*(u_2)\} \end{aligned} \tag{16}$$

In order that this correlation function may depend only on $\xi = x_1 - x_2$ and $\tau = t_1 - t_2$, which we shall assume to be true, the ensemble average under the integral sign in Eq. (16) must have the form

$$\begin{aligned} &E\{ dF(\beta_1, u_1)\, dF^*(\beta_2, u_2)\, dG(u_1)\, dG^*(u_2)\} \\ &= S_p(\beta_1, u_1)\, \delta(\beta_1 - \beta_2)\, \delta(u_1 - u_2)\, d\beta_1\, d\beta_2\, du_1\, du_2 \end{aligned} \tag{17}$$

Substitution of (17) into (16) results in

$$R_p(\xi, \tau) = \iint_{-\infty}^{\infty} e^{i(\beta u\tau - \beta\xi)}\, S_p(\beta, u)\, d\beta\, du \tag{18}$$

We now apply a Fourier transformation to obtain the fixed-frame frequency spectrum

$$\Phi_p(\xi,\ \omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} R_p(\xi,\ \tau)\ e^{-i\omega\tau}\ d\tau$$

$$= \int_{-\infty}^{\infty} \frac{1}{|u|}\ e^{-i\omega\xi/u}\ S_p\left(\frac{\omega}{u},\ u\right)\ du \tag{19}$$

Clearly Eq. (19) is a generalization of Eq. (4).

To compare Eqs. (19) and 13), the latter is Fourier-transformed to yield

$$\frac{1}{2\pi}\int_{-\infty}^{\infty} \overline{\Phi}_p(\omega,\ \xi)\ e^{i\xi\alpha}\ d\xi$$

$$= \overline{\Phi}_p(0,\ \omega)\ \Psi\ (\alpha - \omega/U_c) \tag{20}$$

where

$$\Psi\ (v) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \psi\ (\xi)\ e^{i\xi v}\ d\xi \tag{21}$$

Therefore,

$$\overline{\Phi}_p(\xi,\ \omega) = \overline{\Phi}_p(0,\ \omega)\int_{-\infty}^{\infty} \Psi\ (\alpha - \omega/U_c)\ \ e^{-i\alpha\xi}\ d\alpha \tag{22}$$

Letting $\alpha = \omega/u$ we obtain

$$\overline{\Phi}_p(\xi,\ \omega) = \overline{\Phi}_p(0,\ \omega)\int_{-\infty}^{\infty} \frac{|\omega|}{u^2}\ \Psi\ \left(\frac{\omega}{u} - \frac{\omega}{U_c}\right)\ e^{-i\omega\xi/u}\ du \tag{23}$$

Then equating Φ_p and $\overline{\Phi}_p$ we find a formula to compute $S_p\ (\omega/u,\ u)$ as follows:

$$S_p(\omega/u,\ u) = \left|\omega/u\right|\ \Psi\ (\omega/u - \omega/U_c)\ \overline{\Phi}_p(0,\ \omega)$$

$$= \left|\frac{\omega}{2\pi u}\right| \overline{\Phi}_p(0,\ \omega)\int_{-\infty}^{\infty} \psi(\xi)\ \exp[\ i\xi(\omega/u - \omega/U_c)]\ \ d\xi \tag{24}$$

The frequency cross-spectrum for the structural response can be obtained by a similar superposition. Thus by a generalization of Eq. (12),

$$\Phi_w(x_1, x_2; \omega) = \int_{-\infty}^{\infty} \frac{1}{|u|} H(x_1, \omega/u)\, H^*(x_2, \omega/u)\, S_p(\omega/u, u)\, du \tag{25}$$

Or, letting $k = \omega/u$,

$$\Phi_w(x_1, x_2; \omega) = \int_{-\infty}^{\infty} \frac{1}{|k|} H(x_1, k)\, H^*(x_2, k)\, S_p(k, \omega/k)\, dk \tag{26}$$

Now since

$$S_p(k, \omega/k) = |k| \overline{\Phi}_p(0, \omega)\, \Psi\,(k - \omega/U_c)$$

we obtain a very simple result

$$\Phi_w(x_1, x_2; \omega) = \overline{\Phi}_p(0, \omega) \int_{-\infty}^{\infty} H(x_1, k)\, H^*(x_2, k)\, \Psi\,(k - \omega/U_c)\, dk \tag{27}$$

An Example

As an example, the theory will now be applied to an infinite beam shown in Fig. 1. The beam is backed on the lower side by a space of depth d which is filled with an initially quiescent fluid of density ρ_2 and sound speed a_2. On the upper side the beam is exposed to the excitation of a supersonic boundary-layer turbulent pressure p. The fluid on the upper side of the beam which carries the turbulence has a free stream velocity U_∞, density ρ_1 and sound speed a_1.

As the beam responds to the excitation its motion will generate additional pressures in the fluid media on the upper and lower sides. Denoting these generated pressures by p_1 and p_2, respectively, the governing equation of the beam motion is given by

$$EI \frac{\partial^4 w}{\partial x^4} + m \frac{\partial^2 w}{\partial t^2} = p + (p_1 - p_2)_{z=0} \tag{28}$$

For the purpose of determining the "wave-number response function" $H(x, k)$ the turbulent pressure p should be replaced by $\exp[\, i(\omega t - kx)]$ and

the structural response w equated by $H(x, k)\exp(i\omega t) = A(k)\exp(-ikx)$ $\exp(i\omega t)$. Furthermore, we shall make the usual approximation that p_1 can be calculated without regard to the presence of the turbulence. Then p_1 is governed by the equation

$$\left(\frac{\partial}{\partial t} + U_\infty \frac{\partial}{\partial x}\right)^2 p_1 - a_1^{\,2}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}\right) p_1 = 0 \tag{29}$$

and subject to the conditions that p_1 can propagate only in the positive z direction, and that

$$\left(\frac{\partial p_1}{\partial z}\right)_{z=0} = \rho_1 \left(\frac{\partial}{\partial t} + U_\infty \frac{\partial}{\partial x}\right)^2 w.$$

The solution for p_1, when evaluated at z=0, is known to be[5]

$$(p_1)_{z=0} = -i\rho_1 a_1 \frac{k(\omega/k - U_\infty)^2}{[(\omega/k - U_\infty)^2 - a_1^{\,2}]^{\frac{1}{2}}} A(k)\, e^{-ikx}\, e^{i\omega t} \tag{30}$$

The pressure generated on the lower side of the beam is governed by the equation

$$\frac{\partial^2 p_2}{\partial t^2} - a_2^{\,2}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}\right) p_2 = 0 \tag{31}$$

and subject to the conditions

$$\frac{\partial p_2}{\partial z} = 0 \quad \text{at } z = -d$$

and

$$\frac{\partial p_2}{\partial z} = \rho_2 \ddot{w} \text{ at } z=0$$

The solution for p_2, when evaluated at z=0, is given by

$$(p_2)_{z=0} = \rho_2 \omega^2 \frac{\cot\gamma d}{\gamma} A(k)\, e^{-ikx}\, e^{i\omega t} \tag{32}$$

where

$$\gamma^2 = \left(\frac{\omega}{a_2}\right)^2 - k^2$$

Equations (30) and (32) can now be substituted into Eq. (28) to find A(k), recalling that p must be replaced by exp[i(ωt - kx)] and w by A(k) $e^{-ikx}\ e^{i\omega t}$. The result may be expressed as

$$A(k) = 1/Q$$

where

$$Q = EIk^4 - m\omega^2 + i\rho_1 a_1 \frac{k(\omega/k - U_\infty)^2}{[(\omega/k - U_\infty)^2 - a_1{}^2]^{\frac{1}{2}}} + \rho_2\omega^2 (\cot\gamma d)/\gamma \qquad (33)$$

Thus,

$$H(x, k) = \exp(-ikx)/Q \qquad (34)$$

We are now ready to compute the structural response spectrum due to the excitation of a convected decaying turbulence. For the input spectrum of the turbulent pressure, Eq. (13), we shall use a form proposed by Maestrello[4], for which

$$\psi(\xi) = \exp\left(- \frac{|\xi|}{\alpha\delta} \right)$$

where δ is the boundary-layer thickness and α is an experimentally determined quantity. Corresponding to this ψ function we have

$$\Psi\ (k - \omega/U_c) = \{\pi\alpha\delta\ [(\alpha\delta)^{-2} + (k - \omega/U_c)^2]\}^{-1}$$

Having determined H(x, k) and $\Psi\ (k - \omega/U_c)$ the cross-spectrum of the structural response may be computed using Eq. (27). The integration over k must be carried out, however, on a digital computer.

Figure 2 shows the computed results for the frequency spectrum (i.e., when $x_1 = x_2$) of the structural response using the following physical data:

properties of the beam

EI (bending rigidity) = 3.945×10^4 N-m^2

m (mass per unit length) = 9.746 kg/m

properties of the surrounding fluid media

$\rho_1 = \rho_2 = \rho$ (density) = 0.11015 kg/m^3

$a_1 = a_2 = a$ (speed of sound) 261.6 m/sec

U_∞ (free stream velocity on upper side of beam) = 575.6 m/sec

U_c (convection velocity of the turbulence) = $0.75\ U_\infty$

d(cavity depth) = 0.1178 m

properties of the turbulent pressure [4]

$$\psi(\xi) = \text{decay factor} = \exp\left(-\frac{|\xi|}{\alpha\delta}\right)$$

$$\overline{\Phi}_p(0,\omega) = \text{spectral density} = \frac{\delta}{2U_\infty}\sum_{n=1}^{4} A_n\, e^{-K_n(\omega\delta/U_\infty)}$$

δ(boundary-layer thickness) = 0.279 m

and experimentally determined constants

$\alpha = 3$

$A_1 = 4.4 \times 10^{-2}$	$K_1 = 5.78 \times 10^{-2}$
$A_2 = 7.5 \times 10^{-2}$	$K_2 = 2.43 \times 10^{-1}$
$A_3 = -9.3 \times 10^{-2}$	$K_3 = 1.12$
$A_4 = -2.5 \times 10^{-2}$	$K_4 = 11.57$

Concluding Remarks

The theory developed herein is applicable to any turbulence forcing field which has a cross-spectral density of the form of Eq. (13). It is particularly useful in dealing with boundary-layer turbulence for which the decay in correlation is much more significant than that of the atmospheric turbulence as far as structural response is concerned. For this reason we have chosen an example to illustrate the application of the theory which includes the effect of a cavity and the effect of the free stream velocity. These are main features in the problem of fuselage panel vibration under the excitation of boundary-layer turbulence. The infinite unsupported beam is perhaps the simplest structural model possible which still allows consideration of these features. The advantage of a simple model is to avoid the burdensome mathematical details and concentrate on basic principles.

A better representation of fuselage panels can be obtained by adding evenly spaced elastic supports to the infinite beam. The model then becomes a periodic structure; i.e., a structure which is composed of identical sub-units and for which analytical studies have been carried out extensively[6,7,8,9,10,11]. The elastic supports give rise to multiple reflections and the solution becomes considerable more complicated. Details of this solution will be reported in another paper[12].

Further extensions to the two-dimensional case are obvious. With small modifications the solution for the periodic beam can be changed to suit the case of a row of panels simply supported along two parallel edges. In the sense of Levy's series solution to plate problems involving two simply

supported parallel edges, the solution is still mathematically exact. However, if the structural model has more than one row of panels then only approximate solutions are possible since the spatial variables now cannot be separated. Although new concepts are not required in treating the two dimensional problems the machine computation time can increase astronautically and such studies may best be carried out in the industry.

References

1. Corcos, G. M., "Resolution of Pressure in Turbulence," Journal of Acoustical Society of America, Vol. 35, No. 2, February, 1963, pp. 192-199.

2. Bull, M. K., "Properties of the Fluctuating Wall-Pressure Field of a Turbulent Boundary Layer," University of Southampton Report, A.A.S.U. 234, March 1963 (Also AGARD Rep. No. 455, 1963).

3. Willmarth, W. W. and Wooldridge, C. E., "Measurements of the Fluctuating Pressure at the Wall Beneath a Thick Turbulent Boundary Layer," University of Michigan Report 02920-1-T, April 1962 (Also J. Fluid Mech. Vol. 14, 1962, p. 187).

4. Maestrello, L., Monteith, J. H., Manning, J. C. and Smith, D. C., "Measured Response of a Complex Structure to Supersonic Turbulent Boundary Layers," AIAA 14th Aerospace Science Meeting, Washington, D. C., January 26-28, 1976.

5. Morse, P. M. and Ingard, K. U., Theoretical Acoustics, McGraw-Hill Book Co., 1968, p. 707.

6. Lin, Y. K. and McDaniel, T. J., "Dynamics of Beam-Type Periodic Structures," Journal of Engineering for Industry, Transaction ASME, Vol. 91(9). 1969, pp. 1133-1141.

7. Vaicaitis, R. and Y. K. Lin, "Response of Finite Periodic Beam to Turbulence Boundary-Layer Pressure Excitation," AIAA Journal, Vol. 10, 1972, pp. 1020-1024.

8. Lin, Y. K., "Random Vibration of Periodic and Almost Periodic Structures," in Mechanics Today, Chapter III, ed. Nemat-Nasser, S., Pergamon Press, 1976, pp. 93-123.

9. Mead, D. J., "Free Wave Propagation in Periodically Supported, Infinite Beams," Journal of Sound and Vibration, Vol. 11, 1970, pp. 181-197.

10. Mead, D. J. and Pujara, K. K., "Space-Harmonic Analysis of Periodically Supported Beams: Response to Convected Random Loading, "Journal of Sound and Vibration, Vol. 14, 1971, pp. 525-541.

11. Mead, D. J. "Vibration Response and Wave Propagation in Periodic Structures," Journal of Engineering for Industry, Transaction ASME Vol. 93(B), 1971, pp. 783-792.

12. Lin, Y. K., Maekawa, S., Nijim, H., and Maestrello, L., "Response of Periodic Beam to Supersonic Boundary-Layer Pressure Fluctuation," International Union of Theoretical and Applied Mechanics (IUTAM) Symposium on Stochastic Dynamics, July 19-23, 1976, University of Southampton, England.

List of Figure Captions

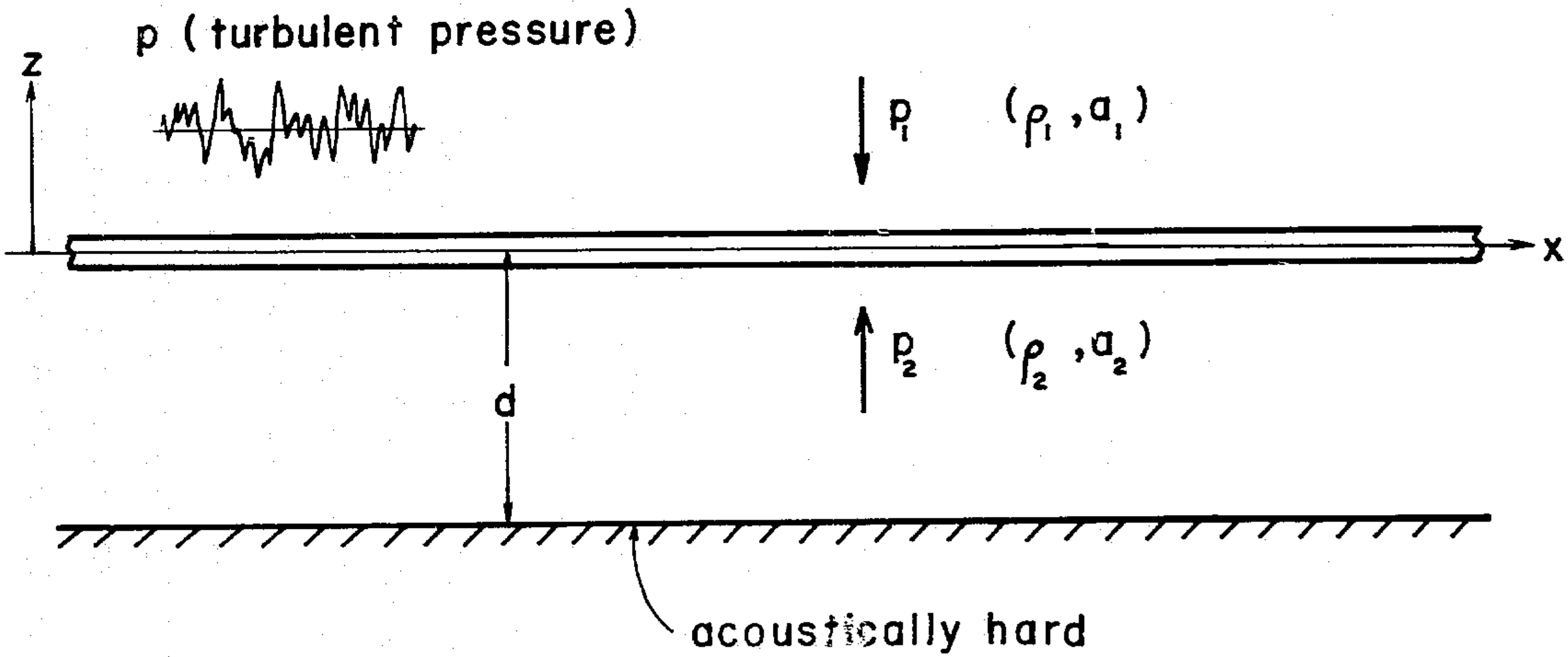

Fig. 1 An infinite beam under the excitation of boundary-layer turbulence

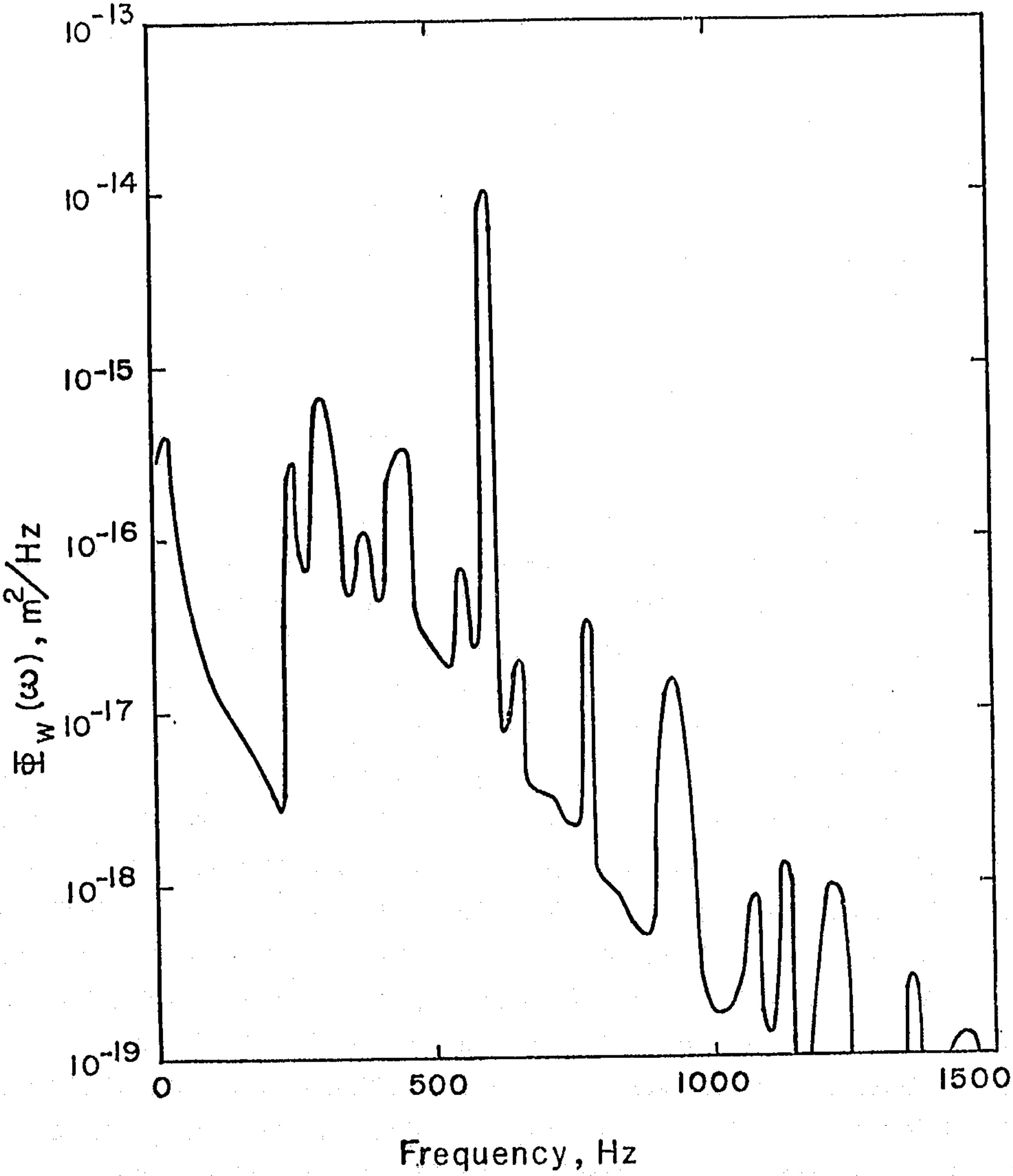

Fig. 2 Spectral density of structural response